**Inequality 3**

1. Prove that is true for all positive numbers a, b, and c with equality only if .

Let be positive numbers.

with equality holds only if

Similarly, with equality holds only if

with equality holds only if

Hence

with equality holds only if .

2. Solve .

3. Solve .

**Method 1**

Consider: . Since

The curve cannot cut the axis and is above the x-axis since .

for all .

**Method 2**

Hence from ,.

4. Solve for .

Since both sides are positive, we square both sides,

This is a strict inequality, we only have

5. (a) Find the solution of the general symmetric inequality:

(b) Hence find where .

(a) By triangular inequalities (1) becomes: Note also, (1) becomes: In general, (1) has solution: , if .

(b) Now we go back to (2) obviously is not the same as the symmetric form as in (1). Put , we get ( is the mid point of and ) Apply previous result,

6. Solve .

We assume is real.

Since both sides are positive, squaring gives

Since , multiple both sides by ,

Hence

7. Prove by mathematical induction the inequality for all .

Let

For , is obviously true.

Assume is true for some , that is

For ,

, by (1)

,

,

,

is true.

By the principle of mathematical induction, is true for all

**1/2/2020**

**Yue Kwok Choy**